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QUANTUM FLUCTUATIONS IN THE NEW INFLATIONARY UNIVERSE

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The evolution of quantum fluctuations of a scalar field in de Sitter space is analyzed in the context of the new inflationary scenario. The duration of the inflationary phase is estimated and the problem of density perturbations resulting from quantum fluctuations of the Higgs field is discussed.

1. Introduction

The inflationary cosmological model [1] assumes a first-order phase transition in which the universe supercools and passes through a de Sitter period of exponential expansion. If the inflationary phase is sufficiently long, this model can give a natural solution to the horizon, flatness and monopole problems. Initially it was not clear how to end inflation and get back to a radiation-dominated universe. Assuming a "normal" first-order phase transition occurring through bubble nucleation and coalescence, Guth [1] and Guth and Weinberg [2] have shown that it leads to a strongly inhomogeneous picture of the universe. Recently, Linde [3] and Albrecht and Steinhardt [4] have suggested that this problem can be avoided in Coleman-Weinberg type models, in which the barrier in the effective potential disappears or becomes very small as the temperature goes to zero. At some temperature T_* the false vacuum is destabilized by thermal or quantum fluctuations, and the Higgs field ϕ starts "rolling" down the effective potential while the universe keeps expanding exponentially. The Coleman-Weinberg potential is very flat near the origin, and the rollover time can be much greater than the expansion time, H^{-1}, where H is the Hubble constant of the de Sitter space. It is expected that the initial scale of spatial variation of the Higgs field is $\sim H^{-1}$, so that one can talk about fluctuation regions of initial size $\sim H^{-1}$. If the rollover time is greater than $\sim 50\,H^{-1}$, then the whole visible universe is contained in one fluctuation region. This new version of Guth cosmology is often called the new inflationary scenario.

Destabilization of the false vacuum in the new inflationary universe, with quantum and gravitational effects taken into account, has been discussed in refs. [5–8]*. It has been shown that the symmetric state is destabilized much earlier than one would naively expect, due to anomalous behavior of fluctuations of a massless minimally coupled field in de Sitter space. For small values of ϕ, the one-loop Coleman-Weinberg effective potential in de Sitter space has the form [11, 10]

$$V(\phi)=\tfrac{1}{2}m^2(T)\phi^2-\tfrac{1}{4}\lambda\phi^4, \tag{1.1}$$

where $m(T)\approx AgT$ is the temperature-dependent effective mass due to interaction with gauge bosons, g is the gauge coupling, $\lambda=Bg^4\ln(\sigma/H)$, $\phi=\sigma$ is the true minimum of the effective potential, A and B are model-dependent constants. At the one-loop level, the symmetric state $\langle\phi\rangle=0$ remains quasistable at all nonzero temperatures. However, when $m(T)$ becomes sufficiently small, higher-order corrections become important.

The effective Higgs mass at $\langle\phi\rangle=0$ equals

$$m_{\rm eff}^2=\langle V''(\phi)\rangle=m^2(T)-3\lambda\langle\phi^2\rangle. \tag{1.2}$$

The last term in eq. (1.2) is the contribution of the Higgs field fluctuations. The infinities in $\langle\phi^2\rangle$ can be absorbed by renormalizing the mass, $m=m(T=0)$, and the conformal parameter, ξ. The renormalized values of m and ξ are assumed equal to zero. In flat space-time $\langle\phi^2\rangle\approx\frac{1}{12}T^2$ and $\lambda\langle\phi^2\rangle$ is always much smaller than $m^2(T)$. However, in de Sitter space the behavior of $\langle\phi^2\rangle$ is quite different. As long as the fluctuations are not too large, one can use perturbation theory and calculate $\langle\phi^2\rangle$ for a noninteracting field ($\lambda=0$). Assuming that the de Sitter stage is preceded by a radiation-dominated Robertson-Walker expansion** and representing $\langle\phi^2\rangle$ as a sum of thermal and vacuum contributions,

$$\langle\phi^2\rangle=\langle\phi^2\rangle_{\rm T}+\langle\phi^2\rangle_{\rm v}, \tag{1.3}$$

one obtains [5, 8]

$$\langle\phi^2\rangle_{\rm T}\approx\frac{T^2}{12}+\frac{H^2}{8\pi Ag} \tag{1.4}$$

and [6, 7]

$$\langle\phi^2\rangle_{\rm v}\approx H^3t/4\pi^2+CH^2. \tag{1.5}$$

* Hawking and Moss [9] have considered a model in which the Higgs field has a small mass, $m>0$, so that the false vacuum remains quasistable down to zero temperature and decays by quantum tunneling. The rollover time τ in the Hawking-Moss scenario has been estimated in ref. [10]. There it is shown that although τ depends on m, it cannot be made greater than $\sim(\alpha H)^{-1}$, where $\alpha=g^2/4\pi$, g is the gauge coupling.

** Other beginnings of the de Sitter stage have also been discussed in the literature [15].

Here, $C \sim 1$ is a constant and time t is set equal to zero at the beginning of the de Sitter stage. The temperature decreases like $\exp(-Ht)$, so that $\langle\phi^2\rangle_T$ approaches a constant, while $\langle\phi^2\rangle_s$ grows (This unusual behavior of $\langle\phi^2\rangle_s$ is explained in sect. 3.) On the other hand, $m^2(T)$ decreases like $\exp(-2Ht)$, at some point m^2_{eff} becomes negative and the symmetric state is destabilized. For typical values of the parameters, this happens when $\langle\phi^2\rangle$ is not much different from H^2.

The purpose of this paper is to analyze the evolution of quantum fluctuations after the false vacuum is destabilized. Initially the fluctuations will grow according to eq. (1.5). When $\langle\phi^2\rangle$ grows sufficiently large, self-interaction becomes important, and the behavior of $\langle\phi\rangle^2$ is modified. Finally, at some point a classical description of the field ϕ becomes appropriate, and the following evolution is described by the classical field equation.

Eq. (1.5) has been obtained in refs. [6, 7]. The derivation of ref. [6] is rigorous but rather complicated, while that of ref. [7] is too sketchy. Since eq. (1.5) is important for the new inflationary scenario, I think it is worthwhile to give a simple derivation of this equation, which will emphasize its physical meaning.

To include the effect of self-interaction, one has to calculate $\langle\phi^2\rangle$ as a function of time for the self-interacting field ϕ in de Sitter space. Needless to say, this problem is not tractable exactly even in flat space-time, and one has to resort to approximate methods. An expansion in powers of λ is not very interesting, since it breaks down when the fluctuations become large. We shall do a little better than that and calculate $\langle\phi^2\rangle$ in a self-consistent approximation, which amounts to a summation of an infinite set of "cactus" diagrams (cactus approximation).

The paper is organized as follows The self-consistent approximation is introduced in sect. 2. Sect. 3 reviews the known results concerning quantum fluctuations for a free scalar field in de Sitter space and gives a new derivation of some results using less rigorous but more intuitive methods These methods are used in sect. 4 to analyze the evolution of quantum fluctuations in the new inflationary universe. In the same section, I estimate the rollover time τ. The result is in a qualitative agreement with the estimation obtained by Linde [7]. Sect. 5 discusses the problem of cosmological density fluctuations produced by the quantum fluctuations of the Higgs field ϕ The results are summarized and discussed in sect 6.

2. Self-consistent approximation

We shall consider a quantum field ϕ described by the lagrangian

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 - V(\phi), \tag{2.1}$$

in de Sitter space. The absolute minimum of $V(\phi)$ is at $\phi = \sigma \gg H$. The metric of the de Sitter space can be written as

$$ds^2 = dt^2 - \exp(2Ht)\,d\mathbf{x}^2 \tag{2.2}$$

The radius of the de Sitter horizon is H^{-1}. We shall disregard thermal effects, since they are unimportant after the false vacuum is destabilized* Then the Coleman-Weinberg potential for $\phi \ll \sigma$ is

$$V(\phi) = -\tfrac{1}{4}\lambda\phi^4 \tag{2.3}$$

and the field ϕ satisfies the operator equation

$$\Box\phi - \lambda\phi^3 = 0 \tag{2.4}$$

All information we need about the quantum fluctuations is contained in the Green function

$$G(x, x') = \langle T\phi(x)\phi(x')\rangle, \tag{2.5}$$

which satisfies the Dyson equation

$$\Box G(x, x') + \int \Sigma(x, x'')G(x'', x')\sqrt{-g}\,d^4x'' = -i\delta(x, x') \tag{2.6}$$

Here, $\Sigma(x, x')$ is the self-energy part and the δ-function is normalized so that

$$\int \delta(x, x')\sqrt{-g}\,d^4x' = 1. \tag{2.7}$$

To the first order in the self-coupling λ, Σ is given by the diagram shown in fig 1a:

$$\Sigma_1(x, x') = -3\lambda\langle\phi^2(x)\rangle\delta(x, x'), \tag{2.8}$$

where

$$\langle\phi^2(x)\rangle = G_0(x, x) \tag{2.9}$$

and G_0 is the free field Green function. $\langle\phi^2\rangle$ and Σ_1 are, of course, divergent and should be renormalized As explained in ref. [6], the infinities in Σ_1 can be absorbed by renormalizing the mass m and the conformal parameter ξ of the field ϕ. (The

* The effects of "Hawking temperature" are included in the vacuum part of $\langle\phi^2\rangle$

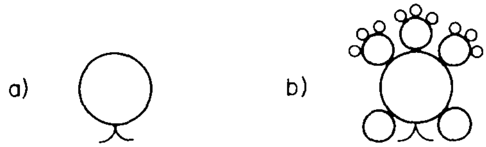

Fig. 1. The self-energy diagrams (a) in the first-order perturbation theory, (b) in cactus approximation.

renormalized values of both m and ξ are assumed to be zero.) Substituting (2.8) in (2.6) we obtain, to first order in λ,

$$\Box G(x, x') - 3\lambda\langle\phi^2(x)\rangle G(x, x') = -i\delta(x, x'). \tag{2.10}$$

To make our approximation self-consistent, we can define $\langle\phi^2\rangle$ in terms of the full Green function G rather than G_0:

$$\langle\phi^2(x)\rangle = G(x, x) \tag{2.11}$$

Eqs. (2.10) and (2.11) with $\langle\phi^2\rangle$ properly renormalized correspond to the summation of an infinite number of "cactus" self-energy diagrams of the form shown in fig. 1b.

The "cactus" approximation defined by eqs. (2.10), (2.11) is analogous to the Hartree approximation used in many-body theory. The same class of diagrams is included in the large-N approximation, which has been studied by a number of authors [12] for a model of N scalar fields ϕ_a with a quartic self-interaction $\lambda(\phi_a\phi_a)^2$. It can be shown that the cactus diagrams give a leading contribution in the limit of large N. For notational simplicity I shall continue to discuss the model of one self-interacting scalar field, the whole analysis can be easily reformulated for the case of an N-component field. For $N = 1$ we have no guarantee that the omitted diagrams are unimportant. However, by including an infinite set of diagrams, we go beyond perturbation theory and one can hope that the results in cactus approximation will reflect the qualitative behavior of the full nonlinear theory.

Instead of calculating the Green function $G(x, x')$ from eqs. (2.10), (2.11), one can use an alternative procedure which I find more illuminating. In the same approximation the field operator $\phi(x)$ can be represented as

$$\phi(x) = (2\pi)^{-3/2}\int d^3k\left[a_k\psi_k(t)e^{ik\cdot x} + \text{h.c.}\right], \tag{2.12}$$

where the mode functions $\psi_k(t)$ satisfy the equation

$$\ddot{\psi}_k + 3H\dot{\psi}_k + k^2e^{-2Ht}\psi_k - 3\lambda\langle\phi^2\rangle\psi_k = 0, \tag{2.13}$$

and $\langle\phi^2\rangle$ is given by

$$\langle\phi^2\rangle = (2\pi)^{-3}\int|\psi_k|^2 d^3k. \tag{2.14}$$

The problem thus reduces to the solution of the coupled equations (2.13), (2.14). A similar system of equations has been used in ref. [10] to calculate the effective potential in de Sitter space in the cactus approximation.

Yet another way of formulating the cactus approximation is in terms of an operator equation,

$$\Box\phi - 3\lambda\langle\phi^2\rangle\phi = 0. \tag{2.15}$$

Here I have omitted the m, ξ and λ renormalization counterterms, which cancel out the divergences in $\langle\phi^2\rangle$.

In the next section we shall discuss the calculation of $\langle\phi^2\rangle$ for a free field ϕ. This corresponds to the first-order approximation for the self-energy part, eqs. (2.8), (2.9). We shall review the results obtained in refs. [5-8], emphasizing the special case $m = \xi = 0$ and rederive some of the results using less rigorous but more intuitive methods. These methods will then be used to analyze the fluctuations in the cactus approximation.

3. A free scalar field in de Sitter space

A free scalar field of mass m and conformal coupling ξ is described by the equation

$$(\Box + m^2 + \xi R)\phi = 0, \tag{3.1}$$

where $R = 12H^2$ is the curvature of the de Sitter space. The effective mass squared of the field is $m^2 + \xi R$. For $m^2 + \xi R > 0$ the theory is stable and for $m^2 + \xi R < 0$ it is unstable (that is, the field has growing modes). We shall pay special attention to the intermediate case, $m^2 + \xi R = 0$, in which the behavior of the theory is very interesting, and which is most relevant for the inflationary scenario. In the following I shall set $\xi = 0$ for simplicity. The dependence on ξ can be recovered by replacing $m^2 \to m^2 + \xi R$ everywhere.

To calculate vacuum averages like $\langle\phi^2\rangle$, in curved space-time, one has to specify what is meant by "vacuum". The choice of a vacuum in curved space-time is not unique, reflecting the fact that there is no unique division into positive and negative frequency modes. However, for a stable theory ($m^2 > 0$) in a de Sitter space, there exists a unique de Sitter-invariant vacuum state, in which all the vacuum averages have the full symmetry of the de Sitter space. Bunch and Davies [13] (see also ref. [14]) have calculated the expectation values of ϕ^2 and of the energy-momentum tensor $T_{\mu\nu}$ in this state. The divergent part of $\langle\phi^2\rangle$ has the form $A + BR$, where A and B are infinite constants. These infinities can be absorbed by the renormalization of m and ξ. In the case of small mass ($m \ll H$), which is of most interest to us, the finite part of $\langle\phi^2\rangle$ in the Bunch-Davies vacuum is [6, 7]

$$\langle\phi^2\rangle_{\mathrm{BD}} = \frac{3H^4}{8\pi^2 m^2} + \mathrm{O}(H^2). \tag{3.2}$$

The vacuum energy-momentum tensor is given by [13]

$$\langle T_{\mu\nu}\rangle_{BD} = \tfrac{1}{4}m^2\langle\phi^2\rangle g_{\mu\nu}. \tag{3.3}$$

If the vacuum state is not de Sitter invariant, it can be shown [6] that $\langle\phi^2\rangle$ and $\langle T_{\mu\nu}\rangle$ approach the de Sitter-invariant values (3.2), (3.3) on a time scale $\sim H/m^2$ and $\sim H^{-1}$, respectively. In other words, all excitations over the Bunch-Davies vacuum are redshifted away.

In the limit $m\to 0$, $\langle\phi^2\rangle_{BD}$ in eq. (3.2) diverges. This infrared divergence arises even in the point-slitted expression $\langle\phi(x)\phi(x')\rangle_{BD}$, indicating that a de Sitter-invariant vacuum state does not exist for a massless, minimally coupled ($\xi = 0$) field. If we set $t = 0$ at the beginning of the de Sitter phase, then it can be shown that, for $t \gg H^{-1}$, $\langle\phi^2\rangle$ is given by [6, 7]

$$\langle\phi^2\rangle = A + H^3t/4\pi^2, \tag{3.4}$$

where A = const.

To understand this unusual behavior of $\langle\phi^2\rangle$, we have to analyze the behavior of the mode functions $\psi_k(t)$ in de Sitter space. For a massless, minimally coupled field, the mode functions are given by

$$\psi_k(t) = \left(\tfrac{1}{4}\pi\right)^{1/2}H\eta^{3/2}\left[c_1H^{(1)}_{3/2}(k\eta) + c_2H^{(2)}_{3/2}(k\eta)\right], \tag{3.5}$$

where $\eta = -H^{-1}e^{-Ht}$, $|c_2|^2 - |c_1|^2 = 1$ and $H^{(1)}_{3/2}(x)$, $H^{(2)}_{3/2}(x)$ are Hankel functions, $H^{(2)}_{3/2}(x) = [H^{(1)}_{3/2}(x)]^* = -(2/\pi x)^{1/2}e^{-ix}(1 + 1/ix)$. The wavelengths of the modes grow like e^{Ht},

$$\lambda = k^{-1}\exp(Ht). \tag{3.6}$$

In flat space-time we would define $\psi_k(t)$ to be positive frequency functions. In an expanding universe, it makes sense to talk about positive and negative frequencies only for the modes with wavelengths much shorter than the horizon, which go through many oscillations during one expansion time. In our case such are the modes with $k|\eta| \gg 1$ and $H^{(2)}_{3/2}(k\eta)$, $H^{(1)}_{3/2}(k\eta)$ are the positive and negative frequency functions, respectively. At the beginning of the de Sitter phase, $t = 0$, $|\eta| = H^{-1}$, and we shall require that the coefficients $c_1(\boldsymbol{k})$ and $c_2(\boldsymbol{k})$ in eq. (3.5) are such that $c_2 \approx 1$ and $c_1 \approx 0$ for $k \gg H$. For $k \lesssim H$, c_1 and c_2 depend on one's assumptions about the initial state of the universe and on the details of the transition to the de Sitter regime.

The crucial observation is that the mode functions (3.5) approach nonzero constants as $t\to\infty$. For $k \gg H$,

$$|\psi_k|^2 = \frac{H^2}{2k^3}\left(1 + \frac{k^2}{H^2}e^{-2Ht}\right). \tag{3.7}$$

We see that the limiting value $H^2/2k^3$ is reached when ke^{-Ht} becomes smaller than

H, that is when the wavelength of the mode becomes greater than the de Sitter horizon, H^{-1}. In the course of expansion, more and more modes come out of the horizon, their contributions add up to $\langle\phi^2\rangle$, and $\langle\phi^2\rangle$ grows unboundedly.

From eqs. (3.2), (3.3) we see that, unlike $\langle\phi^2\rangle$, $\langle T_{\mu\nu}\rangle_{\mathrm{BD}}$ has a finite limit at $m \to 0$:

$$\langle T_{\mu\nu}\rangle_{\mathrm{BD}} \to \frac{3H^4}{32\pi^2} g_{\mu\nu} \tag{3.8}$$

As in the case of $m^2 > 0$, $\langle T_{\mu\nu}\rangle$ in an arbitrary state approaches this limiting value on a timescale $\sim H^{-1}$. This difference between $\langle\phi^2\rangle$ and $\langle T_{\mu\nu}\rangle$ is due to the fact that $\langle T_{\mu\nu}\rangle$ is not sensitive to the contribution of very long wavelength modes.

A rigorous derivation of eq. (3.4) with proper regularization and renormalization of $\langle\phi^2\rangle$ has been given in ref. [6]. Here we shall rederive this equation using simpler and more intuitive methods which, however, treat renormalization in a rather cavalier fashion.

The first approach is to calculate $\langle\phi^2\rangle$ from eq. (2.14) including only the contribution of modes with wavelength greater than the horizon (since we know that these modes are responsible for the growth of $\langle\phi^2\rangle$). This means that the integration over k in (2.14) should be cut at $k \sim He^{Ht}$. Then we can approximately replace $|\psi_k|^2$ under the integral by its limiting value, $H^2/2k^3$:

$$\langle\phi^2\rangle = A + \frac{H^2}{4\pi^2}\int_H^{H\exp(Ht)} k^{-1}\,\mathrm{d}k = A + \frac{H^3 t}{4\pi^2}. \tag{3.9}$$

Here $A = \mathrm{const}$ is the contribution of modes with $k \lesssim H$ (that is of the modes that had wavelengths greater than the horizon at the beginning of the de Sitter phase). The anomalous growth of fluctuations is a special feature of the de Sitter space. If the expansion was slower than exponential before $t = 0$, then the fluctuations at $t = 0$ have no reason to be large, and one can expect that $A \lesssim H^2$.

The time dependence $\langle\phi^2\rangle \propto t$ can be pictured as a brownian motion of the field ϕ. As a result of quantum fluctuations, the magnitude of ϕ on the horizon scale changes by $\pm(H/2\pi)$ per expansion time (H^{-1}). Then the average "displacement" squared is $\langle\phi^2\rangle = (H/2\pi)^2 N$, where $N = Ht$ is the number of "steps".

Eq. (3.4) can also be derived using the operator equation for the field operator ϕ. Let us first consider a massive field,

$$(\Box + m^2)\phi(x) = 0 \tag{3.10}$$

Using this equation we find

$$(\Box + 2m^2)\langle\phi^2\rangle = 2\langle\phi_{,\nu}\phi^{,\nu}\rangle. \tag{3.11}$$

For $m > 0$, $\langle\phi^2\rangle = \mathrm{const}$ and eq. (3.11) gives

$$\langle\phi_{,\nu}\phi^{,\nu}\rangle = m^2\langle\phi^2\rangle. \tag{3.12}$$

In the limit $m \to 0$

$$\langle \phi_{,\nu}\phi^{,\nu} \rangle = 3H^4/8\pi^2, \tag{3.13}$$

where I have used eq. (3.2).

For a massless field ϕ eq. (3.11) is

$$\Box\langle \phi^2 \rangle = 2\langle \phi_{,\nu}\phi^{,\nu} \rangle = 3H^4/4\pi^2 \tag{3.14}$$

or

$$\left(\frac{d^2}{dt^2} + 3H\frac{d}{dt} \right)\langle \phi^2 \rangle = 3H^4/4\pi^2. \tag{3.15}$$

(It is clear from the symmetry that $\nabla\langle \phi^2 \rangle = 0$.) The solution of eq. (3.15) is

$$\langle \phi^2 \rangle = A + B\mathrm{e}^{-3Ht} + H^3 t/4\pi^2, \tag{3.16}$$

in agreement with (3.4).

The two methods just described will be used in the next section to analyze the growth of fluctuations in the cactus approximation.

It is interesting to study the spatial correlations in the field $\phi(\mathbf{x}, t)$. The expectation value of ϕ is $\langle \phi \rangle = 0$, indicating that ϕ can grow in the positive or negative direction with equal probability. However, since the universe expands exponentially, there are strong correlations between the values of ϕ at points separated by distances smaller than $H^{-1}\exp(Ht)$.

The correlation function

$$\Delta(r, t) = \langle \phi(x, t)\phi(x', t) \rangle, \tag{3.17}$$

where $r = |\mathbf{x} - \mathbf{x}'|$, can be written as

$$\Delta(r, t) = (2\pi)^{-3}\int |\psi_k(t)|^2 \mathrm{e}^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')}\mathrm{d}^3k. \tag{3.18}$$

For $k \gg H$, $|\psi_k|^2$ is given by eq. (3.7). Omitting the contribution of modes with $k \lesssim H$, we obtain

$$\Delta(r, t) = (2\pi)^{-2}\int_H^\infty (H^2 + k^2\mathrm{e}^{-2Ht})\frac{\sin kr}{kr}\frac{\mathrm{d}k}{k}. \tag{3.19}$$

The physical distance between the points x and x' is $l = r\exp(Ht)$. For $r \ll H^{-1}$

[that is, for $l \ll H^{-1}\exp(Ht)$]*:

$$\Delta(r,t) = \frac{1}{4\pi^2 l^2} + \frac{H^3 t}{4\pi^2}\left(1 - \frac{1}{Ht}\ln Hl\right). \tag{3.20}$$

The first term in (3.20) is the usual, flat-space, zero-point fluctuation term. We see from eq. (3.20) that for the distances

$$H^{-1} \ll l \ll H^{-1}\exp(Ht) \tag{3.21}$$

and for $Ht \gg 1$ the correlation function is a slowly varying function of l. This means that if we do a measurement of ϕ at two points separated by a distance in the range (3.21), the results will be the same with an accuracy $\sim (Ht)^{-1/2}(\ln(Hl))^{1/2}$. The correlations gradually die out as we go to distances $\sim H^{-1}\exp(Ht)$. In the brownian motion picture, the values of ϕ at points separated by $l \ll H^{-1}\exp(Ht)$ made many brownian steps together and started wandering away from one another only after the co-moving scale l came out of the horizon.

4. Growth of quantum fluctuations

Now we shall use eqs. (2.13)–(2.15) to analyze the growth of quantum fluctuations in our model of a self-interacting scalar field. Suppose the false vacuum is destabilized at $t = 0$ and let us assume for simplicity that $\langle\phi^2\rangle = 0$ at that moment. (This corresponds to neglecting the constant A in eq. (3.4).) Initially $\langle\phi^2\rangle$ will grow according to eq. (3.4). Then its growth will be accelerated by the negative effective mass squared $m^2_{\text{eff}} = -3\lambda\langle\phi^2\rangle$. Let us first consider the initial stage of the evolution when

$$\langle\phi^2\rangle \ll H^2/\lambda, \tag{4.1}$$

so that $|m^2_{\text{eff}}| \ll H^2$. Comparing the last two terms in eq. (2.13) we see that only the modes with wavelengths greater than the horizon ($k \ll He^{Ht}$) are destabilized during this stage, while the higher momentum modes are practically unaffected. Like in the previous section, we shall disregard the contribution of wavelengths shorter than the horizon and write

$$\langle\phi^2\rangle = \frac{1}{2\pi^2}\int_H^{H\exp(Ht)} |\psi_k|^2 k^2\,\mathrm{d}k. \tag{4.2}$$

It can be checked that

$$\psi_k(t) = \psi_k^{(0)}(t)\exp\left\{\frac{\lambda}{H}\int_{t_k}^{t}\langle\phi^2\rangle\,\mathrm{d}t\right\}, \tag{4.3}$$

* Note that for $r \ll H^{-1}$ the contribution of modes with $k \lesssim H$ to $\Delta(r,t)$ is the same constant A that appears in eq (3 9) This contribution is omitted in eqs (3 19), (3 20)

approximately solves the mode equation (2.13), provided that the conditions (4.1) and

$$(\mathrm{d}/\mathrm{d}t)\langle\phi^2\rangle \ll H\langle\phi^2\rangle \tag{4.4}$$

are satisfied*. Here $\psi_k^{(0)}$ is the unperturbed ($\lambda = 0$) mode function and t_k is the time at which the wavelength of the mode becomes greater than the horizon,

$$t_k = H^{-1}\ln(k/H). \tag{4.5}$$

Finally, the integration in (4.2) is taken over modes with $\lambda > H^{-1}$, and we can approximately replace $|\psi_k^{(0)}|^2$ by its limiting value, $H^2/2k^3$.

Then eq. (4.2) gives

$$\langle\phi^2\rangle = \frac{H^2}{4\pi^2}\int_H^{H\exp(Ht)}\frac{\mathrm{d}k}{k}\exp\left\{\frac{2\lambda}{H}\int_{t_k}^{t}\langle\phi^2(t')\rangle\,\mathrm{d}t'\right\}$$

$$= \frac{H^2}{4\pi^2}\int_0^{Ht}\mathrm{d}z\exp\left\{\frac{2\lambda}{H}\int_{H^{-1}z}^{t}\langle\phi^2(t')\rangle\,\mathrm{d}t'\right\}, \tag{4.6}$$

where $z = \ln(k/H)$. Differentiating (4.6) with respect to t, we obtain a differential equation for $\langle\phi^2\rangle$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\phi^2\rangle = \frac{H^3}{4\pi^2} + \frac{2\lambda}{H}\langle\phi^2\rangle^2. \tag{4.7}$$

The solution of this equation with the initial condition $\langle\phi^2\rangle|_{t=0} = 0$ is

$$\langle\phi^2(t)\rangle = \frac{H^2}{2\pi\sqrt{2\lambda}}\tan\chi t, \tag{4.8}$$

where

$$\chi = (\tfrac{1}{2}\lambda)^{1/2}\pi^{-1}H. \tag{4.9}$$

For $\chi t \ll 1$,

$$\langle\phi^2\rangle \approx H^3 t/4\pi^2, \tag{4.10}$$

* The lower limit of integration in eq (4.3) is chosen so that $\psi_k \approx \psi_k^{(0)}$ for $t \sim t_k$, reflecting the fact that the difference between ψ_k and $\psi_k^{(0)}$ is small for modes with wavelengths smaller than the horizon Eq (4.3) applies for $t > t_k$

as expected. The solution (4.8) can also be written as

$$\langle\phi^2\rangle = \frac{H^2}{2\pi\sqrt{2\lambda}}\cot[\chi(t_0 - t)], \tag{4.11}$$

where

$$t_0 = \pi/2\chi = \pi^2(2\lambda)^{-1/2}H^{-1} \tag{4.12}$$

At $t = t_0$ eq. (4.11) diverges, but our approximation breaks down earlier. For $\chi(t_0 - t) \ll 1$ we can write

$$\langle\phi^2\rangle \approx \frac{H}{2\lambda(t_0 - t)}. \tag{4.13}$$

Now it is easily seen that both conditions (4.1) and (4.4) are satisfied if $H(t_0 - t) \gg 1$. From eqs. (4.3) and (4.8) we find the behavior of the mode functions

$$|\psi_k|^2 \approx \frac{H^2}{2k^3}\frac{\cos\chi t_k}{\cos\chi t}, \qquad (t > t_k). \tag{4.14}$$

Eq. (4.7) for $\langle\phi^2\rangle$ can also be derived using the operator equation method. From the operator equation (2.15) we obtain

$$\Box\langle\phi^2\rangle = 2\langle\phi_{,\nu}\phi^{,\nu}\rangle + 6\lambda\langle\phi^2\rangle^2. \tag{4.15}$$

For $\langle\phi^2\rangle \ll H^2/\lambda$, one can expect that $\langle\phi_{,\nu}\phi^{,\nu}\rangle$ is approximately given by the free field expression (3.13), since we know that this quantity is not sensitive to the contribution of long wavelength modes. Then

$$\left(\frac{d^2}{dt^2} + 3H\frac{d}{dt}\right)\langle\phi^2\rangle = \frac{3H^4}{4\pi^2} + 6\lambda\langle\phi^2\rangle^2. \tag{4.16}$$

For $\langle\phi^2\rangle \ll H^2/\lambda$, $d/dt \ll H$, the second derivative can be neglected, and we obtain eq. (4.7).

Although eq. (4.8) becomes inaccurate for large $\langle\phi^2\rangle$, it allows us to estimate the duration of the inflationary phase. The exponential expansion ends when $\langle\phi^2\rangle$ becomes $\sim\sigma^2$, where $\phi = \sigma$ is the true minimum of the effective potential. The approximation leading to eq. (4.8) breaks down for $\langle\phi^2\rangle \gtrsim H^2/\lambda$, when the growth of fluctuations is faster than the expansion, and "rolling down" from $\langle\phi^2\rangle \sim H^2/\lambda$ to $\langle\phi^2\rangle \sim \sigma^2$ takes no longer than $\sim H^{-1}$ (it can be shown that, in the cactus approximation, $\langle\phi^2\rangle$ grows like $(\tau - t)^{-2}$ for $\langle\phi^2\rangle \gg H^2/\lambda$). This means that the rollover time τ can be different from t_0 by no more than H^{-1}, and thus

$$\tau \approx \pi^2(2\lambda)^{-1/2}H^{-1} \tag{4.17}$$

To have enough inflation, we need $H\tau \gtrsim 50$, and so λ must satisfy the constraint

$$\lambda \lesssim 0.02. \tag{4.18}$$

The accuracy of eqs. (4.17), (4.18) depends on the accuracy of the cactus approximation. The cactus approximation is similar to the random phase approximation, since it neglects correlations between different modes. One can expect, however, that at some point correlations become important and the behavior of the field ϕ becomes close to that of a classical field, ϕ_c. A classical behavior of the fluctuations has been assumed by several authors [16–18] who have discussed the density fluctuations in the new inflationary scenario. As long as $\phi_c^2 \ll H^2/\lambda$, the evolution of the classical field is described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\phi_c^2 = \frac{2\lambda}{3H}\phi_c^4, \tag{4.19}$$

which has the solution

$$\phi_c^2 = 3H/2\lambda(t_0' - t), \tag{4.20}$$

where $t_0' = \mathrm{const}$.

The time t_* at which the classical evolution takes over will be estimated in the next section. Here we note that eq. (4.8) is certainly correct up to first order in λ, and one expects that the time t_* should be after the quadratic and higher terms in the expansion of (4.8) in powers of λ become important. For $\chi t \leqslant 1$, the first two terms of this expansion,

$$\langle\phi^2\rangle = \frac{H^3 t}{4\pi^2}\left(1 + \frac{\lambda H^2 t^2}{6\pi^2} + \cdots\right) \tag{4.21}$$

give an accuracy of better than 15%. This indicates that

$$t_* \geqslant \chi^{-1}. \tag{4.22}$$

If we match the solutions (4.8) and (4.20) at $t = t_*$, we find

$$\chi t_0' = \chi t_* + (3/\pi)\tan \chi t_*. \tag{4.23}$$

The rollover time τ equals t_0', and we obtain from eqs. (4.22), (4.23)

$$1.57 < \chi\tau < 1.61. \tag{4.24}$$

This differs from eq. (4.17) by no more than 3%, indicating that eq. (4.17) gives a reasonable estimate of the rollover time.

5. Generation of cosmological density fluctuations

Eq. (4.17) estimates only the *mean* value of the rollover time. The actual value of τ can be different from (4.17) and can fluctuate in space. This is precisely the effect that gives rise to density fluctuations in the new inflationary universe [16–18]. If $\delta\tau_k$ is the fluctuation of τ on a scale characterized by wavenumber k, then at the time when this scale re-enters the horizon [16–18]

$$\delta\rho_k/\rho \sim H\delta\tau_k\,. \tag{5.1}$$

$\delta\tau_k$ can be found from

$$\delta\tau_k \sim \delta\phi_k/\dot{\phi} \tag{5.2}$$

and [16]

$$\delta\phi_k(t) = \left[k^3\Delta_k(t)\right]^{1/2}. \tag{5.3}$$

Here, $\Delta_k(t)$ is the Fourier transform of the two-point correlation function (3.17),

$$\Delta_k(t) = (2\pi)^{-3}\int\Delta(\boldsymbol{x},t)\,\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,\mathrm{d}^3x\,. \tag{5.4}$$

If the evolution of the field ϕ is adequately described by classical field equations, then it can be shown that [16] $\delta\psi_k(t) \propto \dot{\phi}_c(t)$ and τ_k is independent of t. Estimating $\delta\tau_k$ from eqs. (5.2), (5.3) at the time when the galactic scale comes out of the horizon, one finds for that scale $\delta\rho/\rho \sim 50$. This is five or six orders of magnitude too large.

Things look more encouraging in the cactus approximation. The fluctuation of ϕ in this approximation can be written as

$$\delta\phi_k(t) = (k/2\pi)^{3/2}|\psi_k(t)|\,. \tag{5.5}$$

From eqs. (4.8) and (4.14) we see that as $t \to t_0$, the relative fluctuation of ϕ approaches a small constant value,

$$\frac{(\delta\phi_k)^2}{\langle\phi^2\rangle} \to \frac{\sqrt{2\lambda}}{8\pi^2}\cos\chi t_k\,. \tag{5.6}$$

It can be shown that this relation is approximately preserved at late stages of the evolution when $\langle\phi^2\rangle > H^2/\lambda$ (see the Appendix). For $\langle\phi^2\rangle \sim \sigma^2$, $\dot{\phi}$ in eq. (4.26) is $\sim \lambda^{1/2}\sigma^2$ and

$$\delta\tau_k \sim \cos\chi t_k/2\pi\lambda^{1/4}\sigma\,. \tag{5.7}$$

For sufficiently large scales, $\cos \chi t_k \approx 1$ and

$$\frac{\delta\rho_k}{\rho} \sim \frac{H}{2\pi\lambda^{1/4}\sigma}\,. \tag{5.8}$$

With reasonable values of the parameters, eq. (5.8) gives $\delta\rho_k/\rho \sim 10^{-4}$. This is just the value needed for the galaxy formation!

The trouble with this argument is that the cactus approximation neglects correlations between different modes and can hardly be trusted in the analysis of the fluctuations of τ. The time t_* at which the classical description takes over can be estimated in the following way. The field ϕ fluctuates with amplitude $\sim H$ on a timescale $\sim H^{-1}$, and thus the fluctuation of $\dot\phi$ is $\delta\dot\phi \sim H^2$. (This follows also from eq. (3.13)). On the other hand, from eq. (4.19), the classical "velocity" is given by $\dot\phi = \lambda\phi^3/3H$. One can expect that the transition to the classical regime occurs when $\dot\phi$ becomes greater than $\delta\dot\phi$, that is when

$$\langle\phi^2\rangle(t_*) \sim \lambda^{-2/3}H^2\,. \tag{5.9}$$

Assuming that $t_k < t_*$ and evaluating $\delta\tau_k$ at $t \sim t_*$, we find* using eqs. (5.2), (5.6), (5.9) and (4.8)

$$H\delta\tau_k \sim \delta\phi_k(t_*)/H \gtrsim (2\pi)^{-3/2} \sim 0.1\,. \tag{5.10}$$

The density fluctuations are still too large. It is possible to obtain $\delta\rho/\rho \sim 10^{-4}$ for $t_k > t_*$, but this requires ridiculously small values of λ [16–18].

How can we save the inflationary universe? One possibility is to consider N-component Higgs fields with $N \gg 1$, in which case the cactus approximation may be reliable [12]. Perhaps a more attractive alternative is to consider particle models which give a different shape of the effective potential at small ϕ. The authors of refs. [17, 18] have pointed out that the density fluctuations can be made small if the effective potential is sufficiently flat somewhere in the range $H \ll \phi \ll \sigma$.

To illustrate this possibility, suppose that for $\phi \gtrsim \phi_1 \ll \sigma$ the effective potential has the form

$$V(\phi) = -\tfrac{1}{2}\mu^2\phi^2 \tag{5.11}$$

with $\mu^2 \ll H^2$ and $\mu > 0$. A model with $V(\phi)$ like (5.11) has been recently discussed by Dimopoulos and Georgi [19]. We first note that $V(\phi)$ cannot keep the form (5.11)

* Eq. (5.10) assumes that eq. (4.8) applies up to $t \sim t_*$. It is possible, however, that there is an intermediate regime when $\langle\phi^2\rangle$ changes from $\sim H^2/2\pi\lambda^{1/2}$ to $\sim \lambda^{-2/3}H^2$. In that case $H\delta\tau_k$ can be greater than given by eq. (5.10).

up to $\phi \sim \sigma$, since otherwise the false vacuum ($\phi = 0$) energy is $\rho_v \sim \frac{1}{2}\mu^2\sigma^2$ and

$$H^2 = \tfrac{8}{3}\pi G\rho_v \sim \tfrac{4}{3}\pi \frac{\mu^2\sigma^2}{m_P^2} \ll \mu^2 .$$

Here m_P is the Planck mass and I have assumed that $\sigma \ll m_P$. In order to have $\mu^2 \ll H^2$, $V(\phi)$ should have a big dip at $\phi > \phi_1$. (The model of Ref. [19] does not satisfy this requirement.) It will also be necessary to require that

$$\phi_1 \gg H^3/\mu^2 . \tag{5.12}$$

A nice thing about the model (5.11) is that it describes a free field with a tachyonic mass $i\mu$,

$$(\Box - \mu^2)\phi = 0 , \tag{5.13}$$

and the mode functions can be found exactly [13,6]:

$$\Psi_k = \left(\tfrac{1}{4}\pi\right)^{1/2} H\eta^{3/2} H_\nu^{(2)}(k\eta) . \tag{5.14}$$

Here $\eta = -H^{-1}\exp(-Ht)$ and

$$\nu = \left(\frac{9}{4} - \frac{\mu^2}{H^2}\right)^{1/2} \approx \frac{3}{2} - \frac{\mu^2}{3H^2} . \tag{5.15}$$

The asymptotic form of ψ_k at $t \gg t_k$ is

$$|\psi_k(t)|^2 \approx (H^2/2k^3)\exp\left[\frac{2\mu^2}{3H}(t - t_k)\right], \tag{5.16}$$

and analysis similar to that of the previous section gives

$$\langle\phi^2\rangle = \frac{3H^4}{8\pi^2\mu^2}\left[\exp\left(\frac{2\mu^2 t}{3H}\right) - 1\right]. \tag{5.17}$$

We shall use eq. (5.17) assuming that t is sufficiently large, so that the exponential in the square brackets is much greater than one.

The fluctuation of the rollover time, $\delta\tau_k$, can be estimated from

$$\delta\tau_k \sim \delta\phi_k/\Phi , \tag{5.18}$$

where $\delta\phi_k$ is given by eq. (5.5) and $\Phi = (\langle\phi^2\rangle)^{1/2}$.

$$H\delta\tau_k = \left(\frac{3}{2\pi}\right)^{1/2} \frac{H}{\mu} \exp\left(-\frac{\mu^2 t_k}{3H}\right). \tag{5.19}$$

We see that $\delta\rho_k/\rho$ is small if the galactic scale comes out of the horizon at sufficiently late time ($t_k \gg 3H/\mu^2$). If this happens when $\langle\phi^2\rangle \sim \phi_1^2$, then

$$\delta\rho/\rho \sim 0.1\, H^3/\mu^2\phi_1 , \tag{5.20}$$

and the fluctuations have the desired magnitude if $\phi_1 \sim 10^3 H^3/\mu^2$.

If $V(\phi)$ becomes much steeper than (5.11) at $\phi > \phi_1$, the rollover time is the time it takes to get from $\langle\phi^2\rangle = 0$ to $\langle\phi^2\rangle \sim \phi_1^2$:

$$H\tau \sim (3H^2/\mu^2)\ln(\mu\phi_1/H^2) > (3H^2/\mu^2)\ln(H/\mu) , \tag{5.21}$$

where the last inequality follows from the condition (5.12). The inflation is sufficiently large provided that $H \gtrsim 4\mu$.

The conclusion is that, depending on the parameters of the model (5.11), $\delta\rho/\rho$ can take practically any value, including the right one. Other forms of $V(\phi)$ have been suggested in refs. [17, 18, 20]. The effective potential used in refs. [18, 20] is based on the geometric hierarchy model of Dimopoulos and Raby [21]. This model can give small density fluctuations, but probably fails to produce sufficient reheating [20]. In general, to produce small density fluctuations and efficient reheating, the effective potential must have a rather special shape, and it is not clear whether such effective potentials can be naturally obtained in realistic models.

6. Conclusions

In this paper we have discussed the physics of quantum fluctuations in de Sitter space for a simple model of a self-interacting scalar field, eqs. (2.1), (2.3). We have found the average "rollover" time in the new inflationary scenario:

$$\tau = \pi^2(2\lambda)^{-1/2}H^{-1} \tag{6.1}$$

This is in a qualitative agreement with the estimation obtained by Linde [7]. (The difference is only in the numerical coefficient.)

Requiring that the expansion factor, $\exp(H\tau)$, be sufficiently large ($H\tau \gtrsim 50$), we obtain a constraint on λ:

$$\lambda \lesssim 2\times 10^{-2} . \tag{6.2}$$

A warning should be issued that, in general, the results of this paper are not directly applicable to realistic gauge theories, even if the effective potential at small ϕ has the form (2.3). The reason is that the Higgs fields in realistic theories are multi-component fields, and a potential like (2.3) is obtained by choosing some direction in the group space and suppressing other degrees of freedom. This is all right if one is interested in the classical evolution of the field; however, for the analysis of quantum fluctuations one has to include all degrees of freedom.

As an example, let us consider an O(N)-symmetric model

$$L = \tfrac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a + \tfrac{1}{4}\lambda(\phi_a\phi_a)^2, \tag{6.3}$$

where $a = 1,\dots,N$. If we pick some direction in the N-dimensional ϕ-space, say,

$$\phi_a = \phi(1,0,\dots,0), \tag{6.4}$$

then the Lagrangian for ϕ takes the form of eqs. (2.1), (2.3). However, quantum fluctuations occur in all N components of ϕ_a, and the equation corresponding to eq. (4.7) is

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\phi^2\rangle = \frac{H^3}{4\pi^2} + \frac{2\lambda}{3H}(N+2)\langle\phi^2\rangle^2. \tag{6.5}$$

Here $\langle\phi^2\rangle = N^{-1}\langle\phi_a\phi_a\rangle$. The rollover time for this model is

$$\tau = \frac{\pi^2}{H}\left[\frac{3}{2(N+2)\lambda}\right]^{1/2}. \tag{6.6}$$

We note that if N is not too large, eq. (6.1) still gives a reasonably good order-of-magnitude estimate of τ. For $N = 10$, τ changes only by a factor of 0.5 Eq. (6.6) also suggests that taking into account the suppressed degrees of freedom will tend to decrease the magnitude of τ. The single-degree-of-freedom effective potential for the standard SU(5) model has $\lambda \sim 0.5$ [16], and we expect $H\tau \lesssim \pi^2(2\lambda)^{-1/2} \sim 10$, which is not enough.

The analysis of the density perturbations resulting from the quantum fluctuations of ϕ gives, in the cactus approximation, $\delta\rho/\rho \sim 10^{-4}$ – an answer one may be tempted to believe. However, the cactus approximation neglects important correlations between the modes and probably becomes unreliable for large values of $\langle\phi^2\rangle$. The argument of sect. 5 suggests that the classical evolution takes over no later than when $\langle\phi\rangle \sim \lambda^{-2/3}H^2$. This gives $\delta\rho/\rho \gtrsim 0.1$, and thus the density fluctuations are too large. To get around this difficulty, one can consider effective potentials of a different shape. What one needs [17,18] is an effective potential which is sufficiently flat somewhere in the range $H \ll \phi \ll \sigma$ and has a big dip near $\phi = \sigma$. The latter is required for efficient reheating [20].

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Appendix

This appendix discusses the evolution of $\langle\phi^2\rangle$ when $\langle\phi^2\rangle \gg H^2/\lambda$ in the cactus approximation. In this regime the effect of space-time curvature is negligible, and we can use the Minkowsky space d'alambertian in eq. (4.15). The growth of $\langle\phi^2\rangle$ is due to unstable modes with wavelengths $\lambda\!\!\!^{-} > (-m_{\rm eff}^2)^{1/2}$, where $m_{\rm eff}^2 = -3\lambda\langle\phi^2\rangle$. We shall assume that the dominant contribution to $\langle\phi^2\rangle$ is given by the modes with $\lambda\!\!\!^{-} \gg (-m_{\rm eff}^2)^{1/2}$; then

$$\langle\dot{\phi}^2\rangle \gg \langle(\nabla\phi)^2\rangle. \tag{A.1}$$

With this assumption, eq. (4.15) takes the form

$$\frac{d^2}{dt^2}\langle\phi^2\rangle = 2\langle\dot{\phi}^2\rangle + 6\lambda\langle\phi^2\rangle^2. \tag{A.2}$$

Taking the expectation value of the energy conservation law, $T^{\mu}_{0;\,\mu} = 0$, we obtain also

$$\langle\dot{\phi}^2\rangle - \tfrac{3}{2}\lambda\langle\phi^2\rangle^2 = 2E = \text{const}. \tag{A.3}$$

Eqs. (A.2) and (A.3) give the following equation for $\langle\phi^2\rangle$:

$$\frac{d^2}{dt^2}\langle\phi^2\rangle = 9\lambda\langle\phi^2\rangle^2 + 4E. \tag{A.4}$$

At large values of $\langle\phi^2\rangle$ we can neglect the constant $4E$; then eq. (A 4) has a solution

$$\langle\phi^2\rangle = \frac{2}{3\lambda(t_0 - t)^2}. \tag{A.5}$$

In the same approximation, the mode functions satisfy the equation

$$\ddot{\psi}_k = 3\lambda\langle\phi^2\rangle\psi_k = 2(t_0 - t)^{-2}\psi_k, \tag{A.6}$$

which has solutions with $\psi_k \propto (t_0 - t)^2$ and $\psi_k \propto (t_0 - t)^{-1}$. Thus,

$$\psi_k = A_k(t_0 - t)^{-1} + B_k(t_0 - t)^2, \tag{A.7}$$

and

$$\frac{|\psi_k|^2}{\langle\phi^2\rangle} \to \text{const} \qquad \text{as } t \to t_0. \tag{A.8}$$

The weak point of this analysis is the assumption (A.1) which I cannot rigorously justify. I have studied the same problem taking eq. (2.13), (2.14) as a starting point, with an appropriate cutoff in the integral over k. The result is the same, eqs. (A.5) and (A.8). It should be emphasized that eqs. (A.5) and (A.8) apply only in the cactus approximation. As explained in sect. 5, this approximation becomes unreliable at large values of $\langle\phi^2\rangle$.

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