Entanglement and the Conceptual Basis of Quantum Mechanics

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Abstract

Quantum Entanglement has been described by one of the illustrious founders of quantum theory as "the characteristic trait of Quantum Mechanics, the one that enforces its entire departure from classical lines of thought". Quantum Mechanics is generally regarded as the physical theory, which is the best candidate yet for a universal and fundamental description of the physical world. Indeed the transition from classical to quantum physics marks a genuine revolution in our understanding of the physical world.

From the inception of Quantum Mechanics, the concept of measurement has proved to be a permanent source of difficultly. The Einstein-Bohr debate from which resulted the EPR paradox has centered upon this difficulty. The postulate of collapse and the dynamics of the theory seem to be flatly in contradiction with one another.

After a brief introduction of these aspects of Quantum Mechanics and its interpretational basis, we discuss some of the recent developments of the entanglement in relation to quantum systems composed of two identical constituents. Since this seems to be related to the impossibility of attributing a complete set of properties to both the constituent particles, it implies definite constraints on the mathematical form of the state vector associated with the whole system. Clearly a full clarification of entanglement in general and of the role of the identity operator within the quantum mechanical formalism in particular will continue to be the most sought-after aspects of this fundamental description of the physical world,

1. Entanglement

One of the creators of Quantum Mechanics, Werner Heisenberg, wrote in his "**Physics and Philosophy**", '*The idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees, independently of whether or not we observe them ... ...is impossible*'. This has given rise to infinite discussions since the very beginning of this highly successful theory to the extent that another Nobel Laureate, Richard Feynman, in his "**The Character of Physical Law**" was forced to declare that, '*I think I can safely say that nobody understands Quantum Mechanics'*.

The difficulty basically is to understand the collapse of the wave functions, or the measurement problem of standard quantum mechanics. The formal machinery of the theory essentially consists of the eigenvalue equation,

$$A_i\psi_i = \lambda_i\psi_i \text{ ……… } (1)$$

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where A_i represents the operators of the system corresponding to physical observables, ψ_i are the wave functions or eigenstates and λ_i the allowed eigenvalues. Suppose we consider the particular measurement process in which we measure the spin of an electron placed in a magnetic field B (Stern-Gerlach experiment). Let us denote by ϕ the wave function of the electron (the micro system) and let φ be the wave function of the apparatus (the macro system) so that the total wave function of the entire system is,

$$\psi = \phi\varphi \qquad (2)$$

Now with respect to the direction of the magnetic field, the spin of the electron can be up or down so that its wave function is either ϕ_+ or ϕ_-. Similarly φ_{up} and φ_{down} represent the wave function of the apparatus corresponding to it's being in the state indicting spin up or spin down. Initially, the wave function of the complete system may be

$$\psi(t=0) = \phi_+\varphi_o \qquad (3)$$

where, φ_o is the initial state of the apparatus (no reading) and the electron has, say, spin up(with respect to B).

The Schrodinger equation governs the time evolution of the state vector $\psi(t)$, which evolves continuously into the final state (when the electron has left the magnetic field),

$$\Psi(t) \xrightarrow[t\to\alpha]{} \Psi_{out}(t)\,\phi_+\varphi_{up} \qquad (4)$$

Similarly if we begin with $\phi_-\varphi_o$, the corresponding asymptotic state will be

$$\phi_-\varphi_o \rightarrow \phi_-\,\varphi_{down} \qquad (5)$$

But the problem is that we do not know prior to measurement the state of the micro system. The initial state of the electron is a superposition,

$$\Phi_0 = \alpha|\phi_+|^2 + \beta|\phi_-|^2 \qquad (6)$$

with

$$|\alpha|^2 + |\beta|^2 = 1 \qquad (6a)$$

Hence initially we have

$$\psi(t=0) = (\alpha\phi_+ + \beta\phi_-)\varphi_o \qquad (7)$$

Since the Schrodinger equation is linear, the state for the micro and macro systems become "entangled" as

$$\psi_0 = (\alpha\phi_+ + \beta\phi_-)\varphi_o \xrightarrow[t\to\alpha]{} \phi_{out} = \alpha\phi_+\varphi_{up} + \beta\phi_-\varphi_{down} \qquad (8)$$

so that the measuring apparatus is in a superposition of 'up' and 'down' states.

In everyday life we do not see such superpositions and therefore the standard or Copenhagen interpretation of quantum mechanics makes it a principle that there is a discontinuous change at observation i.e.

$$\alpha\phi_{+}\varphi_{up} + \beta\phi_{-}\varphi_{down} \xrightarrow[\text{discontinous}]{} \begin{cases} \phi_{*}\varphi_{up} & \text{with probability} \quad |\alpha|^2 \\ \phi_{-}\varphi_{down} & \text{with probabili ty} \quad |\beta|^2 \end{cases} \qquad (9)$$

This is known as the "reduction" or "collapse" of the wave function. In other words, the state vector suddenly reduces from the superposition of states to the observed state. This is the measurement problem.

Now this quantum entanglement has been considered by Schrodinger as '*the characteristic trait of Quantum Mechanics, the one that enforces its entire departure from classical lines of thought*.' Interestingly enough, this entanglement idea constitutes an essential point in many aspects of quantum information and quantum computation theory. In fact, the possibility of teleportation, of securing private keys in cryptography, of deriving quantum algorithms for solving computational problems all seem to depend on the properties of entangled states.

2. The Macro-Objectification Problem

The standard theory incorporates the wave packet reduction (WPR) as an essential part of the theory i.e. in a measurement process, the reduction occurs and macro- objectification takes place giving rise to either $\alpha\phi_{+}\varphi_{up}$ or $\beta\phi_{-}\varphi_{down}$ with probabilities $|\alpha|^2$ and $|\beta|^2$ respectively. Now-a-days there is a general consensus that this solution is absolutely unacceptable for two basic reasons:

1. It corresponds to assuming that the linear character of the theory is broken at a certain level. In other words, quantum theory is unable to explain how the apparatus behaves in the way it does as is required by the WPR postulate.

2. Even if we assume that quantum mechanics breaks down at the macro level, it is clear that the theory does not contain any criterion for identifying the borderline between macro and micro systems, between linear and non-linear processes, between deterministic and non-deterministic realms of reality.

In the eyes of people like J.S. Bell, there is nothing in the theory, which can characterise the borderline between the classical and quantum aspects of Nature. In the historical debate between Einstein and Bohr, Bohr succeeded in refuting Einstein's criticisms at the Solvay Conferences by pointing out that some macroscopic parts of the apparatus have to be treated fully quantum mechanically. Wigner has transferred the borderline to the gray area between the physical and the conscious mind and Wheeler seems to support some version of the many-world interpretation (MWI) put forward by his student Everett. The MWI is an approach to quantum theory according to which in addition to the world we are in, there are many other similar worlds, which exist in parallel at the same space and time. Since the MWI seems merely to shift the problem from our world to other similar worlds, which we cannot experience, there is very little we can say about the validity of such an interpretation.

Since the early 1970s, philosophers of physics have become painfully aware of the non-locality inherent in standard quantum theory. This arises most dramatically perhaps in the context of the projection postulate, which asserts that upon measurement of physical system its state will collapse or be projected onto one of the possible values of the measured quantity. As we have already stated, this postulate is difficult to accept because what effects this discontinuous change in the physical state of a system is not at all clear. It is especially worrying when applied to entangled compound systems whose components are well separated in space. The classical example is the Einstein-Podolsky-Rosen Gedanken experiment in which two particles initially interacting become well separated in space. Their quantum-mechanical state is entangled in the sense that the collapse resulting from a measurement on one of them simultaneously and instantaneously affects the other. This is action at a distance par excellence.

A possible way out was noticed by van Frassen who in 1972 proposed to eliminate the projection postulate altogether from the theory. Bohm's 1952 theory following de Broglie's 1920 proposal also eliminates the projection postulate as do various many-worlds interpretations. Thus. van Frassen's model interpretation relies in particular on a distinction between the ‘value state’ of a system and the ‘dynamical state’ of the system. The value state describes the system's properties while the dynamical state determines the properties the system might have at a later time. The dynamic state is just the quantum state and it never collapses. The value state is something other then the quantum state, especially when the dynamical state is not a pure state. When the dynamical or quantum state of a system is mixed, the dynamical state contains the possible value states and it is the experiment which determines the value states that are observable. Clearly this seems to be somewhat of a quibble on words but it seems to have been taken seriously by many.

3. The Measurement Problem

The problem of measurement in quantum mechanics arises out of the fact that several aspects of the theory appear to be in conflict with each other. As David Albert has said, ‘*The dynamics and the postulate of collapse are flatly in contradiction with one another ..... The postulate of collapse seems to be right about what happens when we make measurements and the dynamics seems to be bizarrely wrong about what happens whenever we are not making measurements.*’ This has come to be known as the measurement problem.

The measurement problem is not just an interpretational problem; it raises deeper issues as well such as more general philosophical questions, for example Cartesian idea of observation as the creation of ‘inner reflections’ and on the other hand, neo-Kantian conceptions of observations as a quasi- externalized physiological process. In particular, in the EPR paper we find the following criterion for the existence of physical reality:

> *If without in any way disturbing a system we can predict with certainty ... the value of a physical quantity, then there exists an element of physical reality corresponding to this physical reality.*

This criterion, characterises physical reality in terms of objectivity meaning its independence from any direct measurement. When a direct measurement of physical reality occurs it merely passively reflects rather than actively constitutes what is observed. Einstein's realist criterion is opposed to Kantian ‘*veil of perception*’. To be specific according to Kant, rather than simply reflecting an independently existing reality, ‘*appearances*’ are due to the act of perception, which conforms to the fundamental categories of sensible intuition. As Kant wrote, ‘Not only are the drops of rain mere appearances but even their round shape, and even the space in which they fall are nothing in themselves but merely modifications of fundamental forms of our sensible intuition and .... the transcendental object remains unknown to us’.

On the other hand Einstein's realism takes the point of view that when we observe raindrops, we are looking at objects ‘*in themselves*’, existing independently of being perceived. Not only do the raindrops exist independently of our observations but also observing them, what we see is a true picture of how they really are. According to the realist point of view, observations not only reflect the way things are during observation but also before and after the observation.

The Copenhagen school, especially by Bohr and Heisenberg, opposed such realism. Although Bohr was not a Kantian and did not believe that space and time as well as cause and effect had to be taken as a priory category for comprehension of all knowledge, yet he took the position that ‘*measurement has an essential influence on the conditions on which the very definition of the physical quantities in question rests.*’ Thus whereas for Kant concepts played their role prior to experience and give form to what is experienced, for Bohr it was exactly the opposite i.e. objective reality, in particular conditions of observation determine the applicability of concepts.

Like Bohr, Heisenberg was also opposed to Einstein's realism and he took as meaningless all sorts of metaphysical considerations about the ‘*true nature of reality*’. In his words,

> *It is possible to ask whether there is still concealed behind the statistical Universe of perception a 'true' Universe in which the law of causality would be valid. But such speculation seems to us to be without value and meaning, for physics must confine itself to the description of the relationships between perceptions.*

Philosophers have often accused the Copenhagen school for being subjectivistic or positivistic. But anyone who has studied Bohr carefully will agree that his views are neither. There are both realist and anti-realist elements in them and they have affinities to Kant as well as to neo-Kantian philosophy.

It is clear that quantum mechanics is fundamentally about atoms and molecules, about photons and electrons, about quarks and strings but not about those particular macroscopic regularities associated with what we call measurements of the properties of these things. But if these entities are not to be somehow identified with the wave function itself, then where are they to be found in the quantum description?

End of Part I (to be continued)